Pure Mathematics

Ivy Collegiate School

2020-2021

Department:	Mathematics	**Time:**	MTuW 13:20 – 14:20
Email:	mathematics@ivycollegiateschool.org	**Place:**	808.

Course References: This is a list of various interesting and useful books and online resources that were used in the construction of the course. You are not required to purchase any of these, but they may be useful for occasional consultation. (Note: They are available in the restricted library.)

- Terence Tao, *Analysis I/II*, Springer, 3rd eds., 2016.
- Daniel J. Velleman, *How to Prove It: A Structured Approach*, Cambridge University Press, 3rd ed., 2019.
- John B. Fraleigh, *A First Course in Abstract Algebra*, Pearson, 7th ed., 2014.
- Miles Reid and Balazs Szendroi, *Geometry and Topology*, Cambridge University Press, 2005.
- F. William Lawvere and Stephen H. Schaneul, *Conceptual Mathematics: A First Introduction to Categories*, Cambridge University Press, 1997.
- Kenneth A. Ross, *Elementary Analysis: The Theory of Calculus*, Springer, 2nd ed., 2013.
- Vikash Tiwari and V. Seshan, *Pathfinder for Olympiad Mathematics*, Pearson, 2017.
- Dierk Schleicher and Malte Lackmann, *An Invitation to Mathematics: From Competitions to Research*, Springer, 2010.
- Brian Gaulter and Mark Gaulter, *Further Pure Mathematics*, Oxford University Press, 2001.

Objectives: This course is the second advanced course at the post-AP level in the ICS mathematics curriculum. This course surveys a variety of topics in higher mathematics, emphasizing elements of logic and set theory, real analysis, complex analysis, abstract algebra, number theory, and topology. This is a proof-based course that focuses on the development of mathematical argument and research. Students will explore logical statements of truth, forms of mathematical induction, the description of sets, groups and categories, fields and vector spaces, and mathematical structures. Students who are successful in this course will find new directions to inspire their pursuit of mathematics at a higher level. Successful students will be able to read and write formal proofs, analyze mathematical texts for rigor, and solve a variety of Olympiad-level problems. The course requires students to participate in several "proof parties" that encourage interaction and argumentation. This is a challenging course and is only open to those students with a strong background in mathematics at the post-AP level and an interest in further mathematics.

Co-/Prerequisites: Students must have successfully completed Multivariable Calculus, but Linear Algebra & Differential Equations (LADE) is also recommended.

Grading Policy: Homework and quizzes (60%), Finals and Project (40%)

Pure Mathematics Course Outline:

- Week One ..... Introduction to proofs & mathematical reasoning
- Week Two ..... Logic
  - Statements and truth
  - Truth tables
- Week Three ..... Set theory
  - Sets
  - Subsets
- Week Four ..... Abstract algebra
  - Semi-groups
  - Monoids
  - Groups
- Week Five ..... Number theory
  - Ring of integers
  - PROOF PARTY!
- Week Six ..... Real analysis
  - Ring of integers
  - Ordered rings and fields
  - Completeness
- Week Seven ..... Topology
  - The topology of $\mathbb{R}$
  - Open and closed sets
- Week Eight ..... Complex analysis
  - The complex field
  - Basic topology of C
- Week Nine ..... REVIEW
- Week Ten ..... Logic
  - Logical equivalence
  - Validity in sentential logic
  - PROOF PARTY!
- Week Eleven ..... Set theory
  - Relations and functions
  - Equinumerosity
- Week Twelve ..... Abstract algebra
  - Structures and homomorphisms
  - Images and kernels
  - Normal subgroups and ring ideals
- Week Thirteen ..... Number theory
  - Prime numbers
  - GCD and LCM
- Week Fourteen ..... Real analysis
  - Limits and continuity
  - PROOF PARTY!
- Week Fifteen ..... REVIEW
- Week Sixteen ..... Topology
  - Topological spaces
  - Bases
  - Continuous functions and homeomorphisms
- Week Seventeen ..... Complex analysis
  - Complex-valued functions
  - The Riemann Sphere
  - PROOF PARTY!
- Week Eighteen ..... Measure theory
  - Lebesgue measure
- Week Nineteen ..... Measure theory
  - Lebesgue integration
- Week Twenty-Thirty-two ..... SPRING REVIEW PERIOD
- Week Twenty-three-Thirty-seven ..... CAPSTONE RESEARCH
- Week Thirty-eight ..... CAPSTONE PRESENTATIONS

Capstone Project: The Capstone projects are designed to give you the opportunity to showcase (beyond the examination) what you have learned and the skills you have mastered as part of the course. While every project assignment may be different in scope, target, grading, and type of organization, each is intended to require you to "dig deep" and impress us with your creativity. For this set of courses, you are required to submit an original research paper related to a topic you have learned in class, produce a 30-minute video tutorial on a select topic, or develop a programming project related to a concept from the course. Rubrics will be available in the Spring semester.

Important Dates:

Final Examination Semester I ..... Dec 22, 2020
Final Examination Semester II .....Jun 3, 2021
Capstone Deadline ..... TBD

Course Policy:

- A TI-NSpire CX-CAS is recommended.
- Knowledge of MATLAB/Octave and LaTeX is recommended but not required.

Class Policy:

- Regular attendance is essential and expected.
- All assignments should be submitted with complete worked-out solutions.

Academic Honesty: Students are expected to abide by the policies regarding Academic Honesty as laid out in the ICS Student Handbook. Any violations will be forwarded for administrative review and the possible imposition of academic penalties.